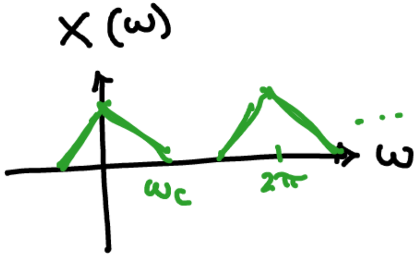
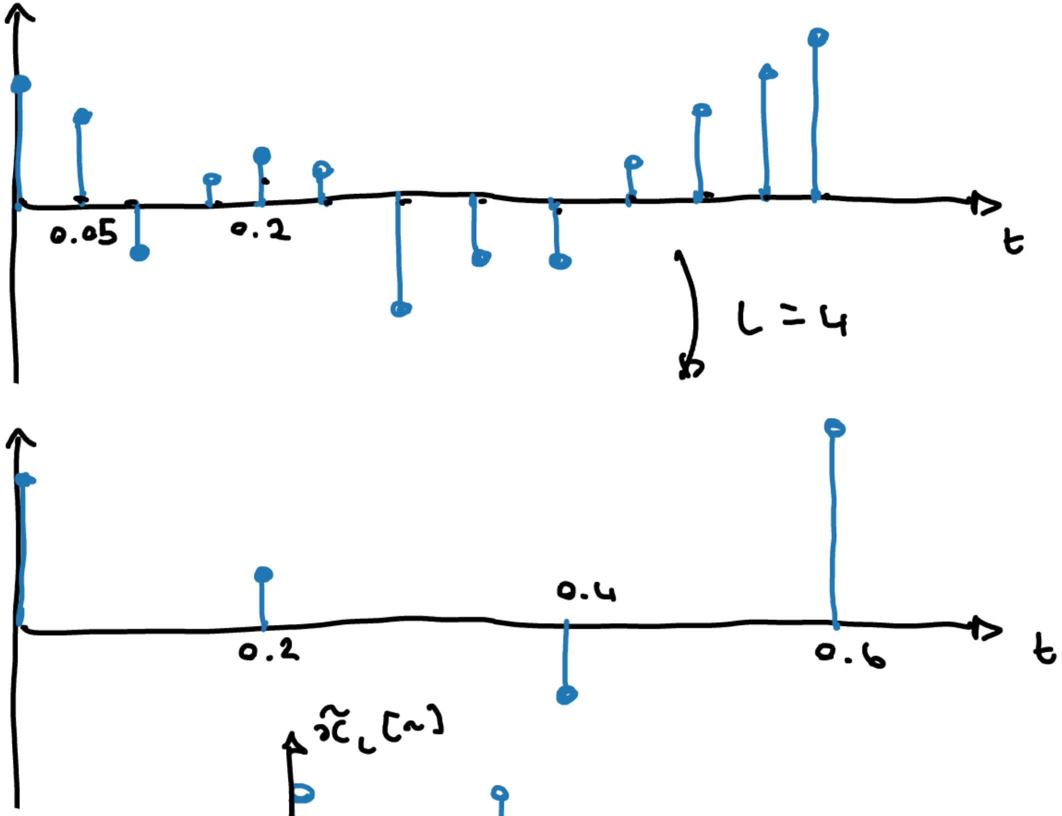
Upsampling and downsampling

Herman Kamper

Downsampling intuition



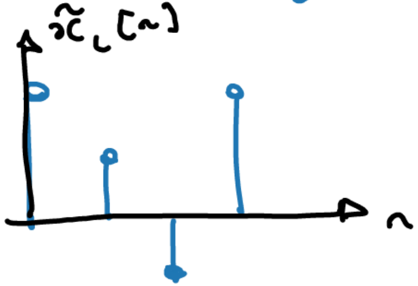
?



$f_s =$ 20 ~~k~~Hz

$f_s =$ 5 ~~k~~Hz

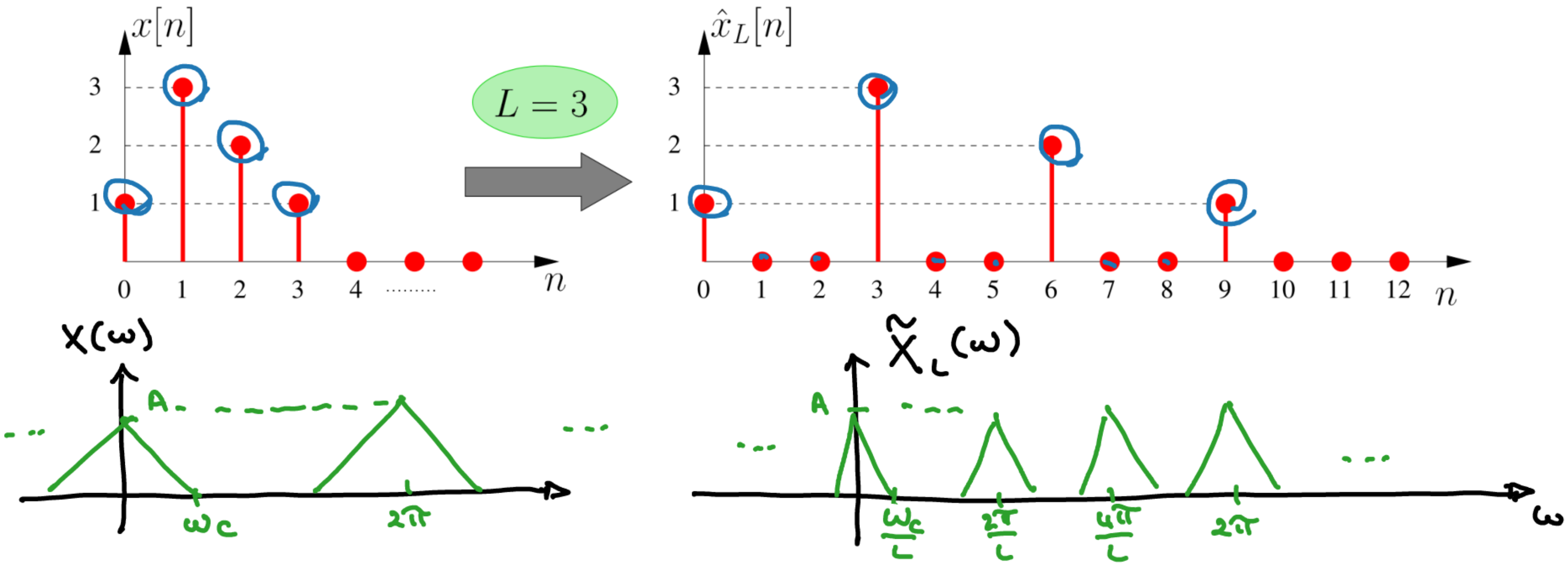
$\tilde{x}_L[n] = x[nL]$



Upsampling

Insert $L-1$ zeros between each sample of $x[n]$:

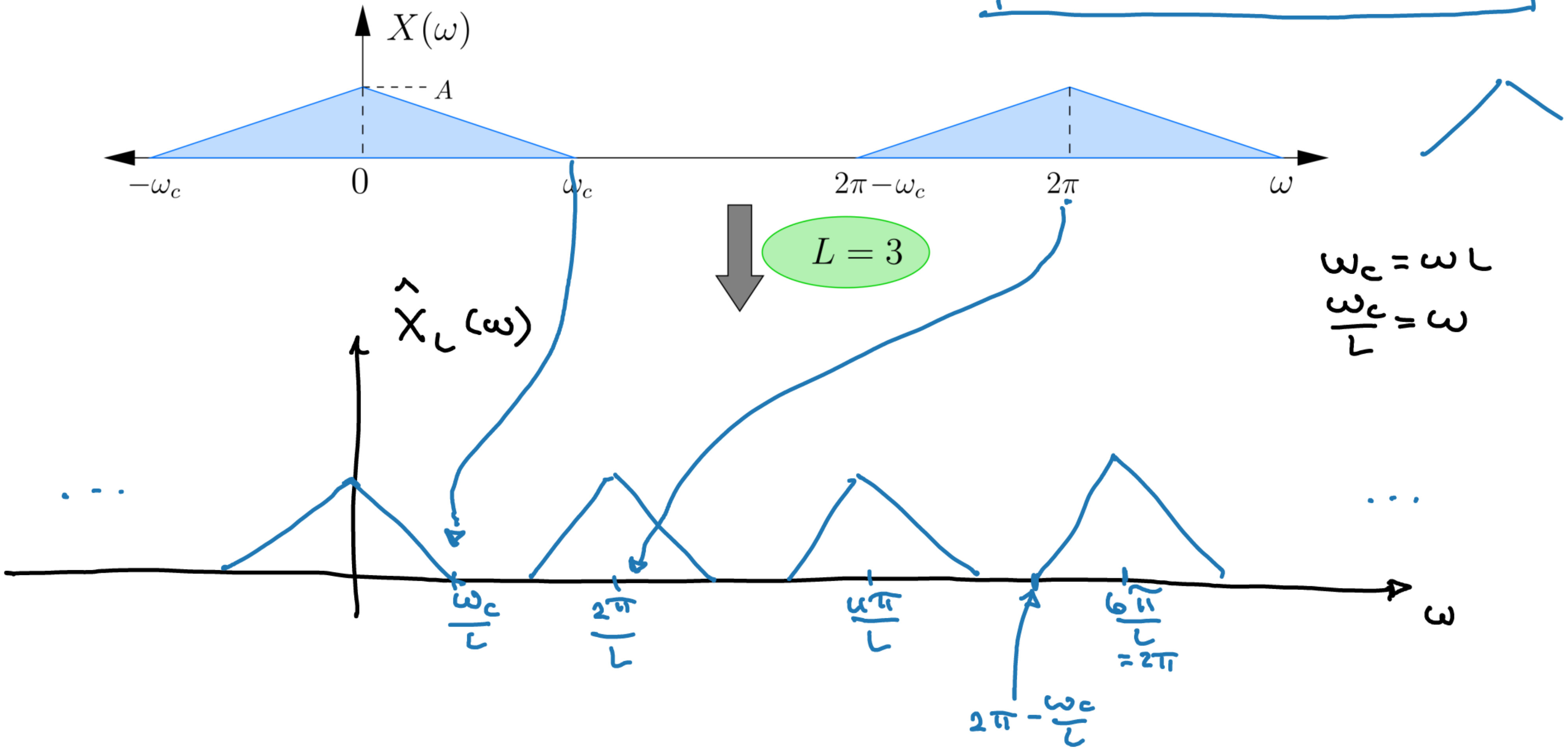
$$\hat{x}_L[n] = \begin{cases} x[n/L] & \text{when } n = kL \\ 0 & \text{otherwise} \end{cases}$$



DTFT:

$$\hat{X}_L(\omega) = \sum_{n=-\infty}^{\infty} \hat{x}_L[n] \cdot e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega L n}$$

$$\boxed{\therefore\ \hat{X}_L(\omega) = X(\omega L)}$$

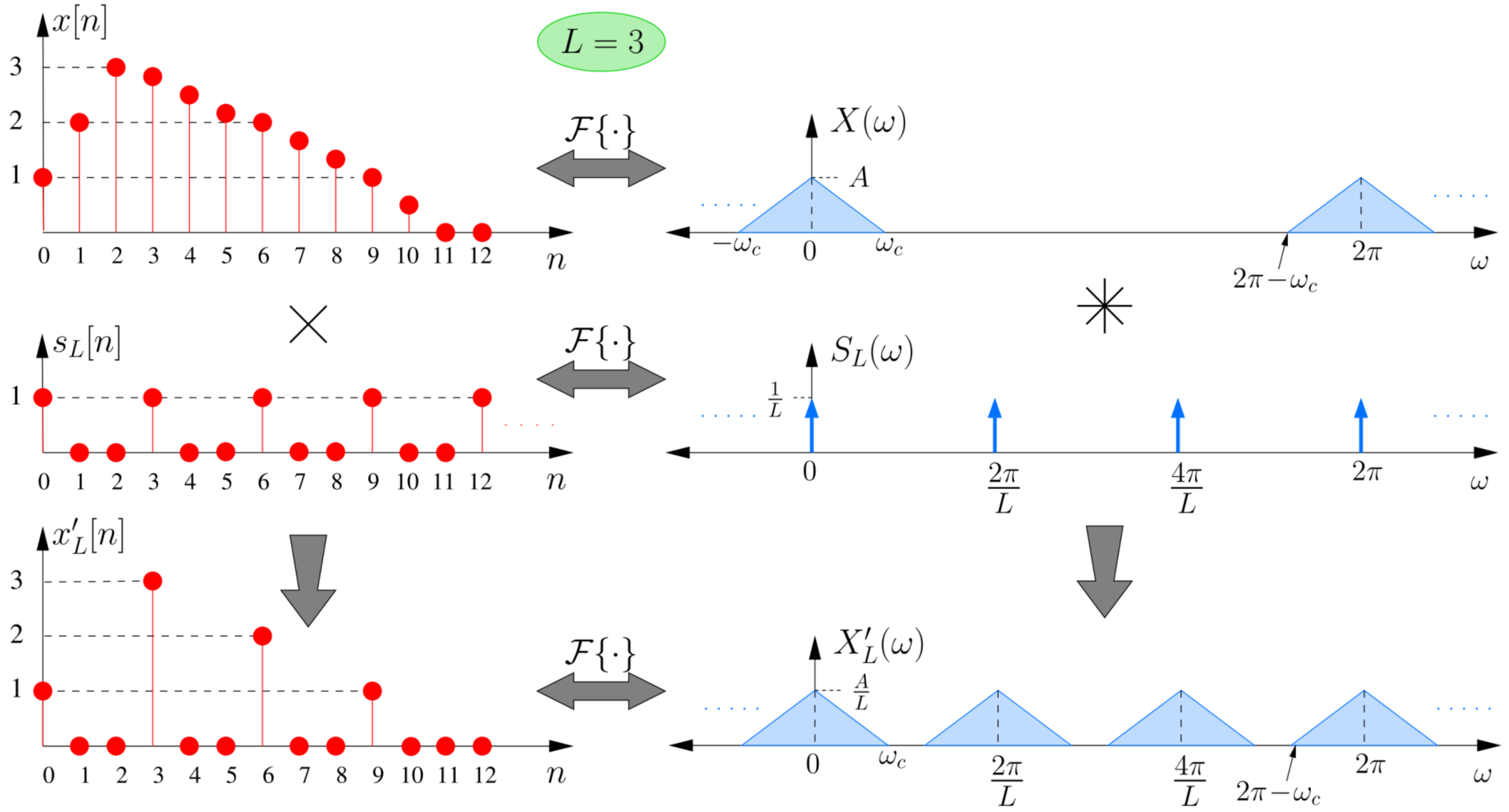


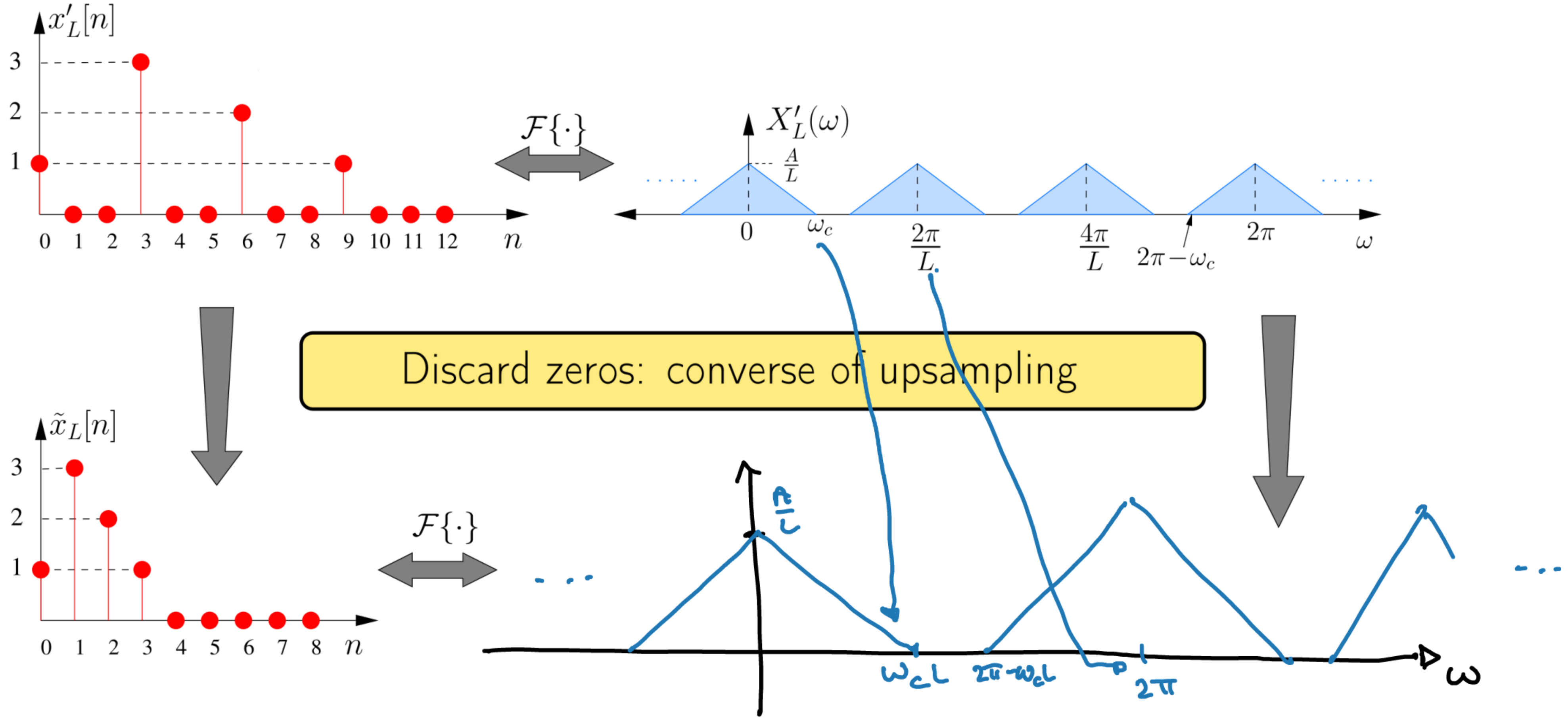
$$\omega_c = \omega L$$

$$\frac{\omega_c}{L} = \omega$$

Downsampling

Keep each Lth sample: $\tilde{x}_L[n] = x[nL]$




$x'_L[n]$
$\mathcal{F}\{\cdot\}$
$X'_L(\omega)$
$\frac{A}{L}$
0 ω_c $\frac{2\pi}{L}$ $\frac{4\pi}{L}$ $2\pi-\omega_c$ 2π ω
Discard zeros: converse of upsampling
$\tilde{x}_L[n]$
$\mathcal{F}\{\cdot\}$
$\frac{A}{L}$
$\omega_c L$ $2\pi-\omega_c L$ 2π ω