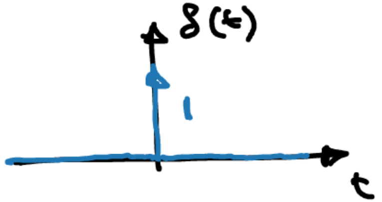
Recap of continuous signal processing

Herman Kamper

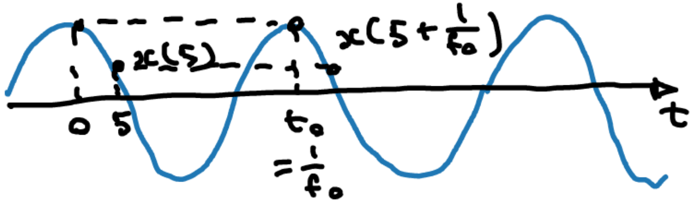
Recap of continuous signal processing

- Continuous signal zoo
  - Dirac delta (impulse)

$$\delta(t) = \begin{cases} 0 & \text{if } t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t) \cdot dt = 1 \end{cases}$$



  - Sinusoidal and exponential signals
- Signal properties ~ Energy, power
  - Periodicity $x(t + t_0) = x(t)$ for all t

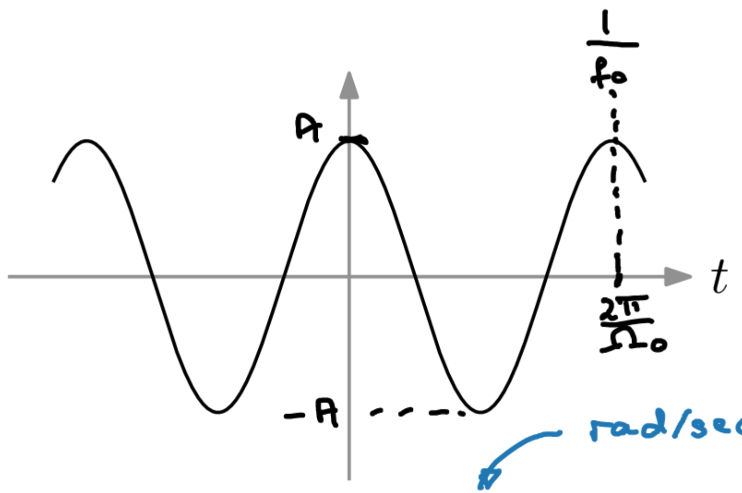


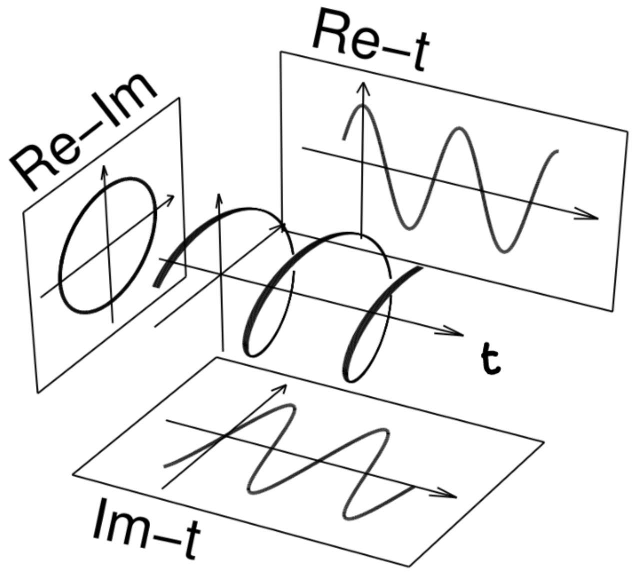
f_0

$t_0 = \frac{1}{f_0}$

  - Even and odd signals
- Operations on signals Stretch, scale, shift
  $\alpha x(t)$
  - Convolution
- Transforms
  - The Fourier transform

Sinusoidal and exponential signals





rad/sec

$$x(t) = A\cos(\Omega_0 t) \qquad 2\pi f_0 = \Omega_0$$

$$= A\cos(2\pi f_0 t)$$

cycles/sec [Hz]

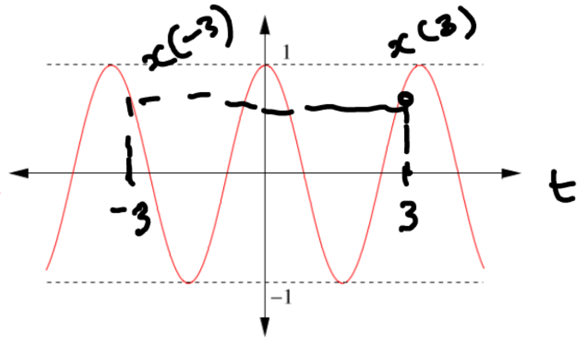
$$= \frac{A}{2} e^{j2\pi f_0 t} + \frac{A}{2} e^{-j2\pi f_0 t}$$

$$x(t) = e^{j\Omega_0 t}$$

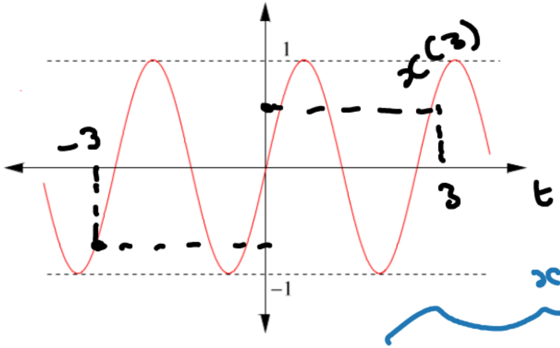
Euler's identity: $e^{j\theta} = \cos(\theta) + j\sin(\theta)$

Even and odd signals

A signal is **even** when $x(-t) = x(t)$



A signal is **odd** when $x(-t) = -x(t)$

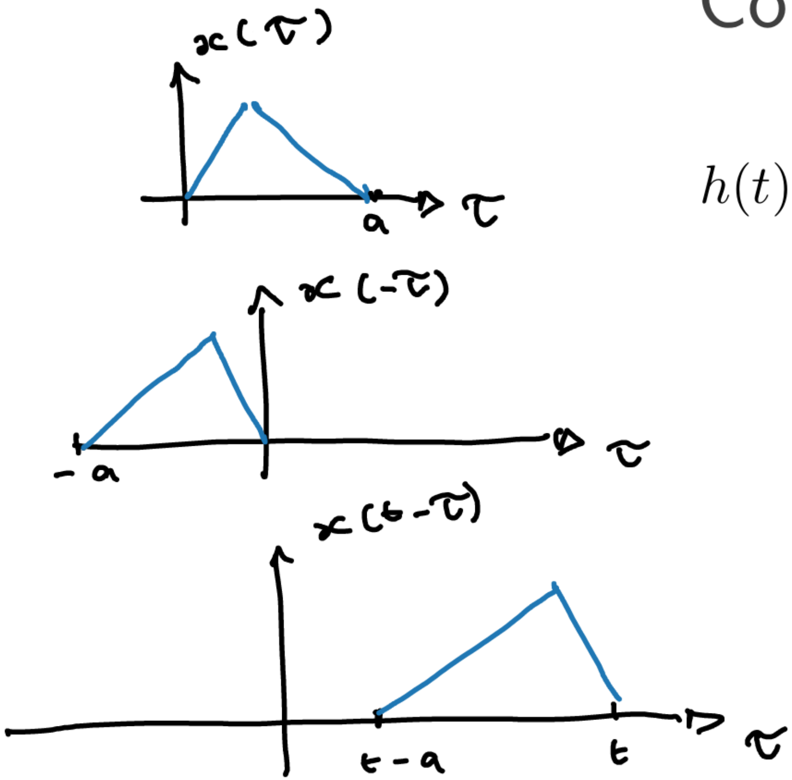


Any signal can be decomposed into even and odd parts:

$$x(t) = \underbrace{\frac{x(t)}{2} + \frac{x(t)}{2}}_{x(t)} + \underbrace{\frac{x(-t)}{2} - \frac{x(-t)}{2}}_{=0}$$

$$= \underbrace{\frac{x(t) + x(-t)}{2}}_{\text{even}} + \underbrace{\frac{x(t) - x(-t)}{2}}_{\text{odd}}$$

Continuous convolution



$$h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau$$

$$= x(t) * h(t)$$

$$t - \tau = a$$
$$\tau = t - a$$

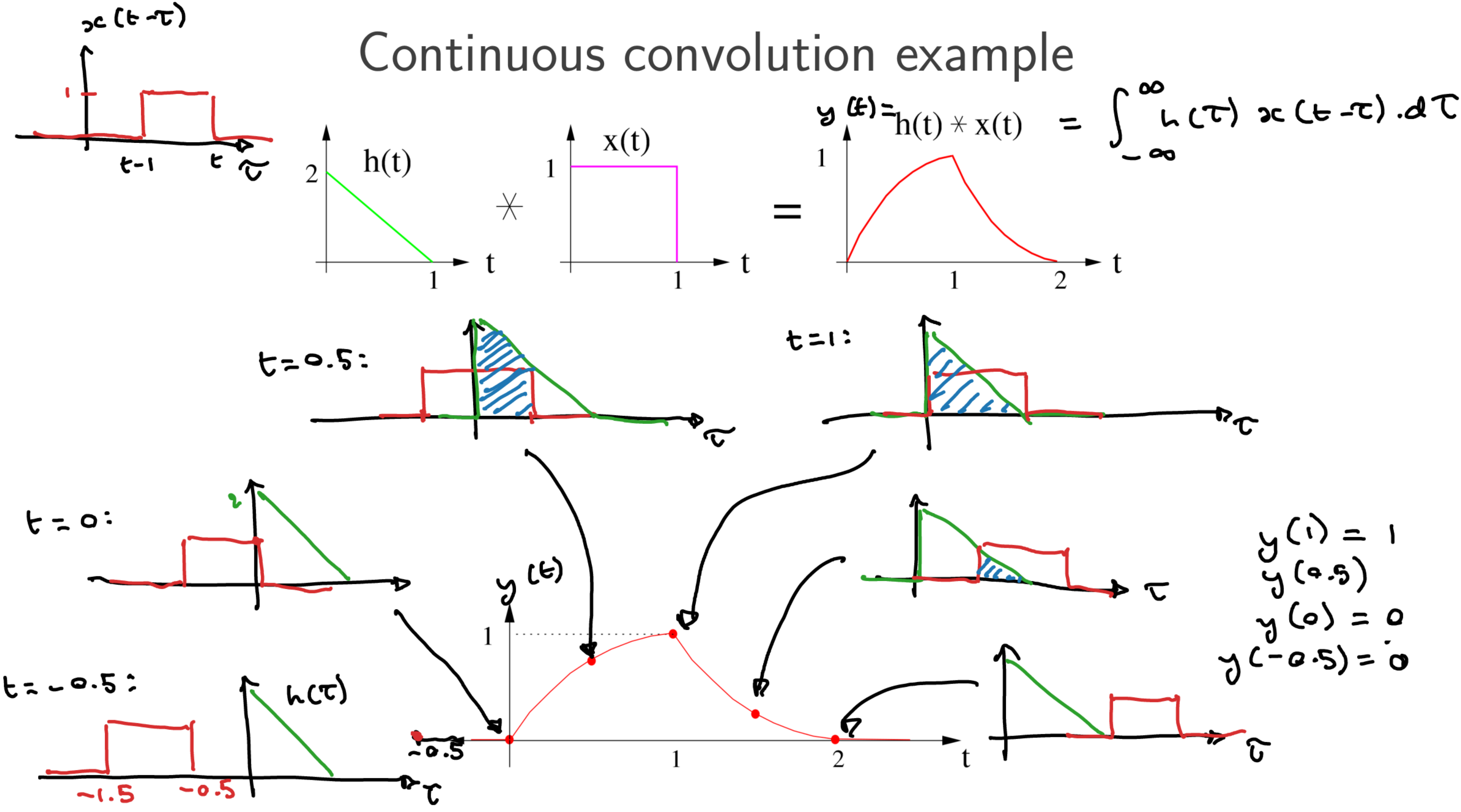
$$t - \tau = 0$$
$$\tau = t$$

LTI:

$x(t) \rightarrow$ [$h(t)$] $\rightarrow y(t) = x(t) * h(t)$

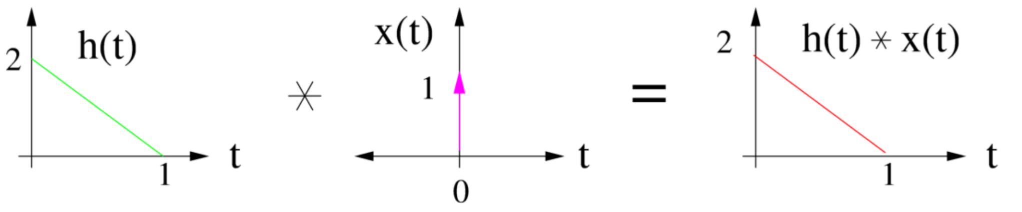
Continuous convolution example

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

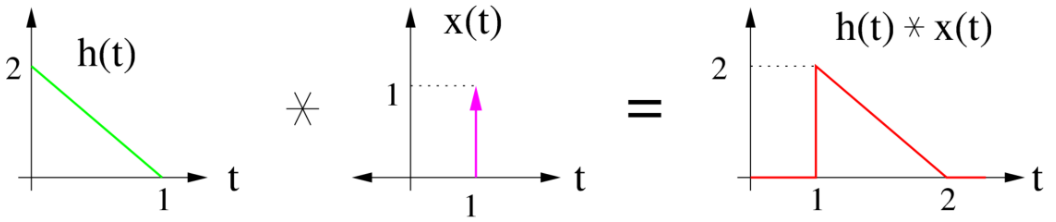


Continuous convolution with impulses

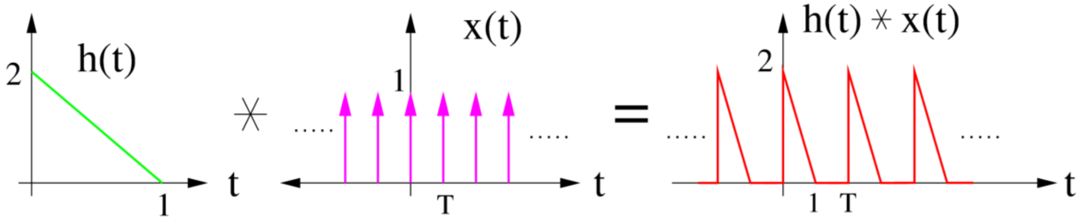
Convolution with single impulse:



Convolution with single shifted impulse:



Convolution with impulse train:

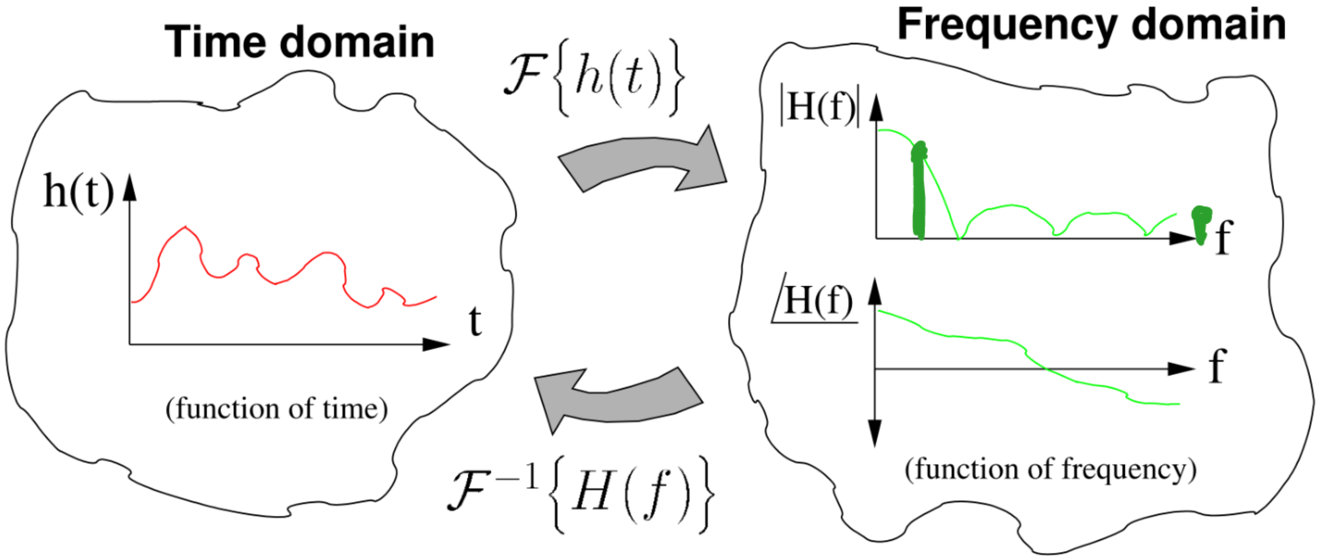



-33

Fourier transform

$$\mathcal{F}\{h(t)\} = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft}\, dt = H(f)$$

$$h(t) = \mathcal{F}^{-1}\{H(f)\} = \int_{-\infty}^{\infty} H(f) \cdot e^{j2\pi ft} \cdot df$$



Properties of the Fourier transform

- Linearity:

$$\mathcal{F}\{\alpha x(t) + \beta y(t)\} = \alpha \mathcal{F}\{x(t)\} + \beta \mathcal{F}\{y(t)\}$$

- Symmetry:

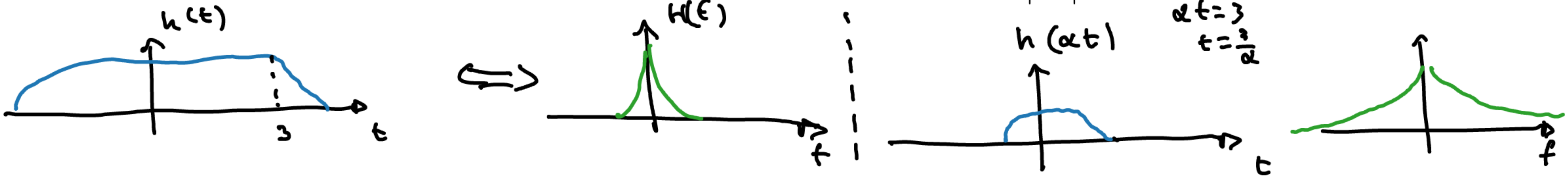
$$\text{if } \mathcal{F}\{h(t)\} = H(f) \text{ then } \mathcal{F}\{H(t)\} = h(-f)$$

- Time-shifting:

$$\mathcal{F}\{x(t - t_0)\} = e^{-j2\pi f t_0} \mathcal{F}\{x(t)\}$$

- Time-frequency scaling:

$$\text{if } \mathcal{F}\{h(t)\} = H(f) \text{ then } \mathcal{F}\{h(\alpha t)\} = \left|\frac{1}{\alpha}\right| H\left(f/\alpha\right)$$



- Convolution:
  - Time-domain convolution corresponds to frequency-domain multiplication:

$$\mathcal{F}\{h(t) * x(t)\} = \mathcal{F}\{h(t)\} \cdot \mathcal{F}\{x(t)\}$$

  - Frequency-domain convolution corresponds to time-domain multiplication:

$$\mathcal{F}\{h(t) \cdot x(t)\} = \mathcal{F}\{h(t)\} * \mathcal{F}\{x(t)\}$$

- Even and odd functions:
  - If $h(t)$ is real, $H(f)$ has even real and odd imaginary parts
  - If $h(t)$ is real and even, $H(f)$ is also real and even:

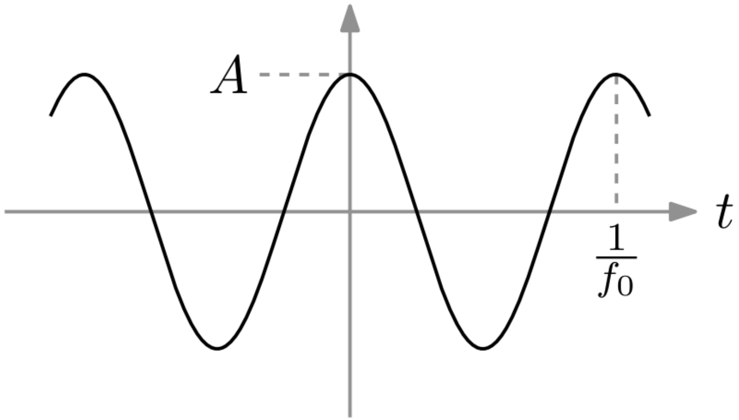
$$\mathcal{F}\{h_e(t)\} = H_e(f) = \int_{-\infty}^{\infty} h_e(t) \cos(2\pi f t) \, dt$$

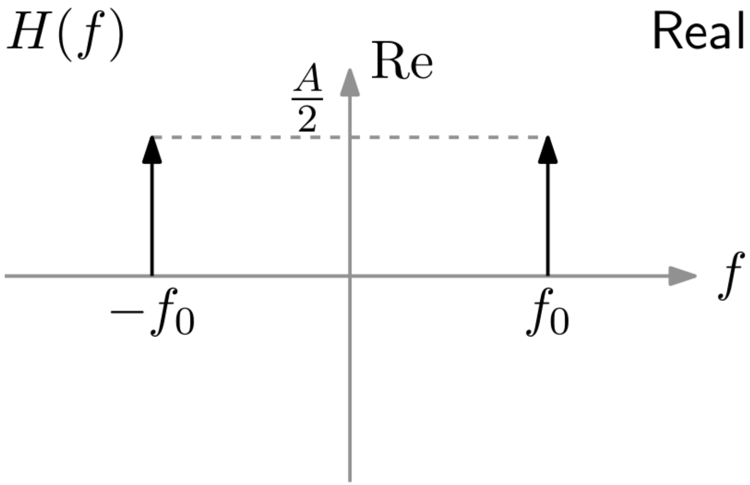
  - If $h(t)$ is real and odd, $H(f)$ is imaginary and odd:

$$\mathcal{F}\{h_o(t)\} = H_o(f) = -j \int_{-\infty}^{\infty} h_o(t) \sin(2\pi f t) \, dt$$

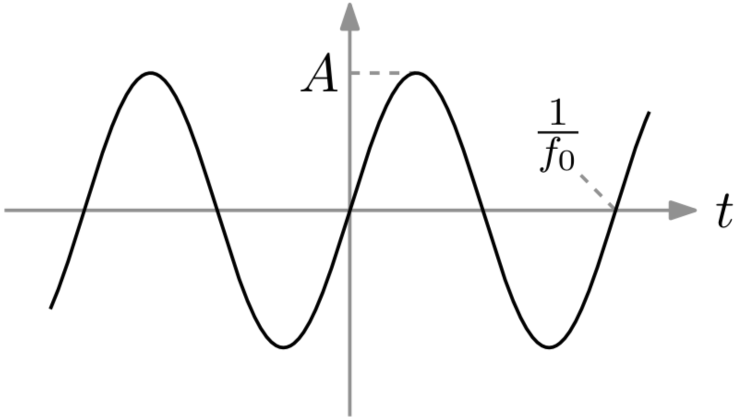
Time domain	Frequency domain

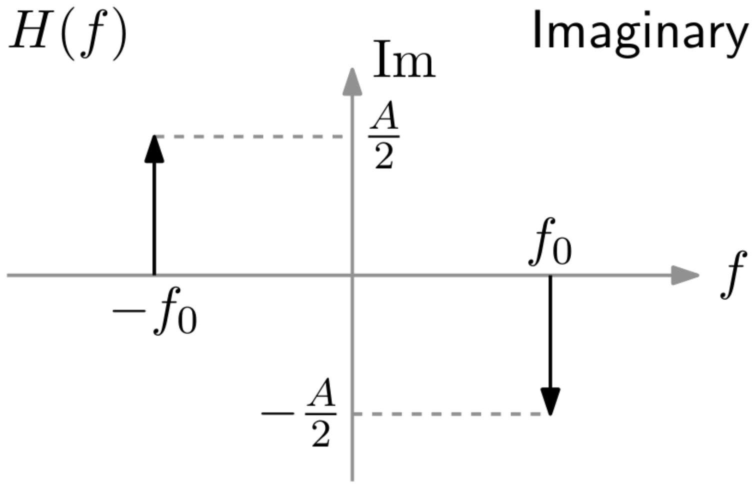
$$h(t) = A\cos(2\pi f_0 t)$$

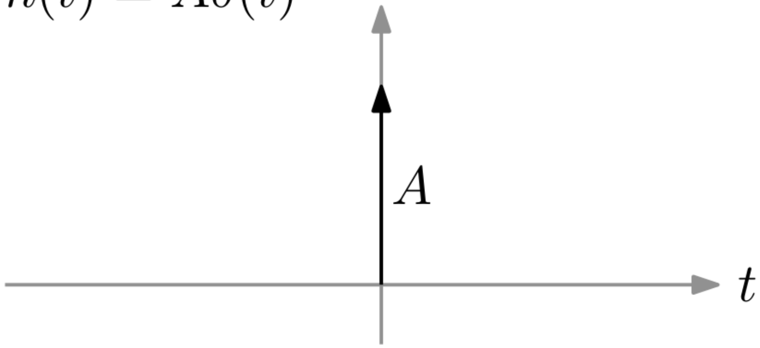
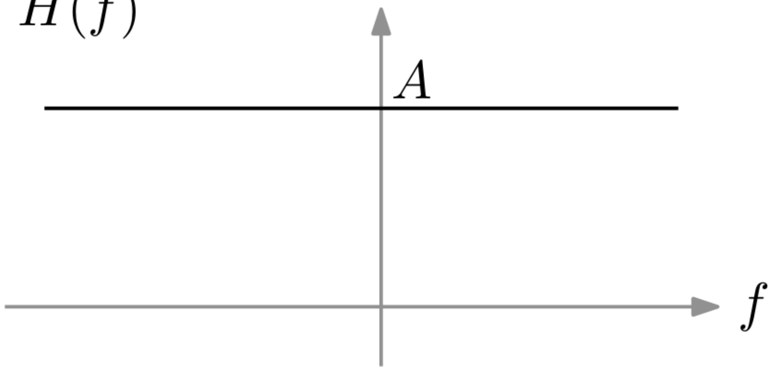
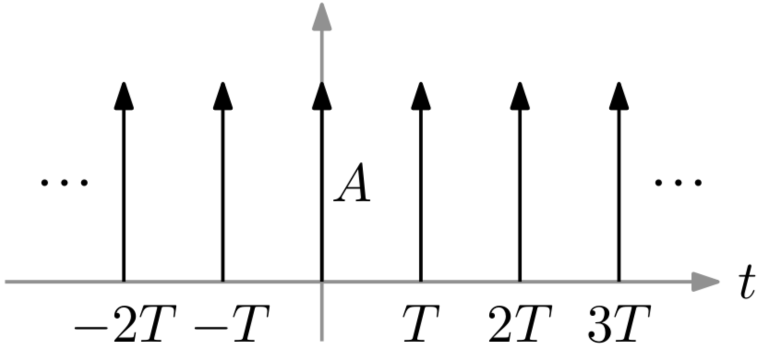
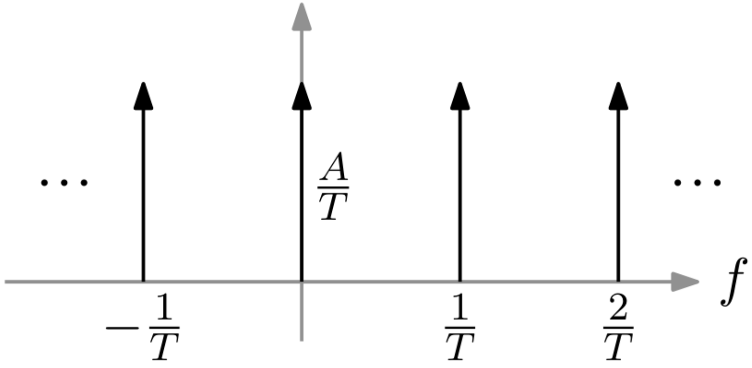


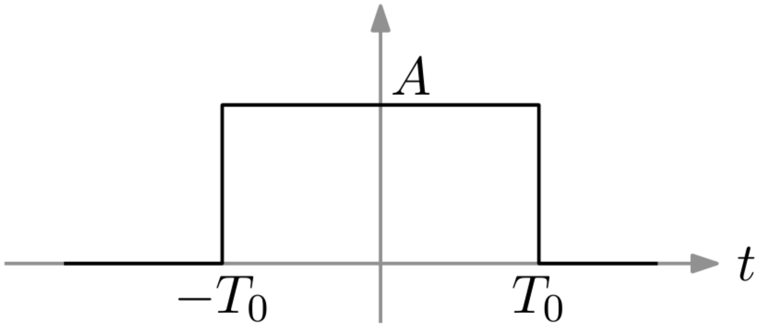
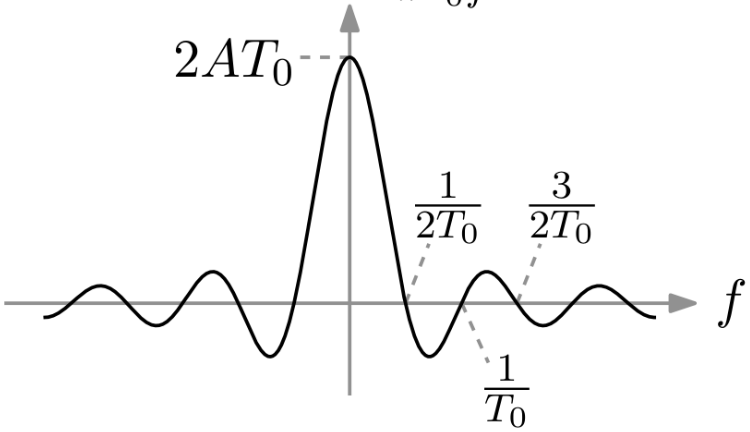


$$h(t) = A\sin(2\pi f_0 t)$$

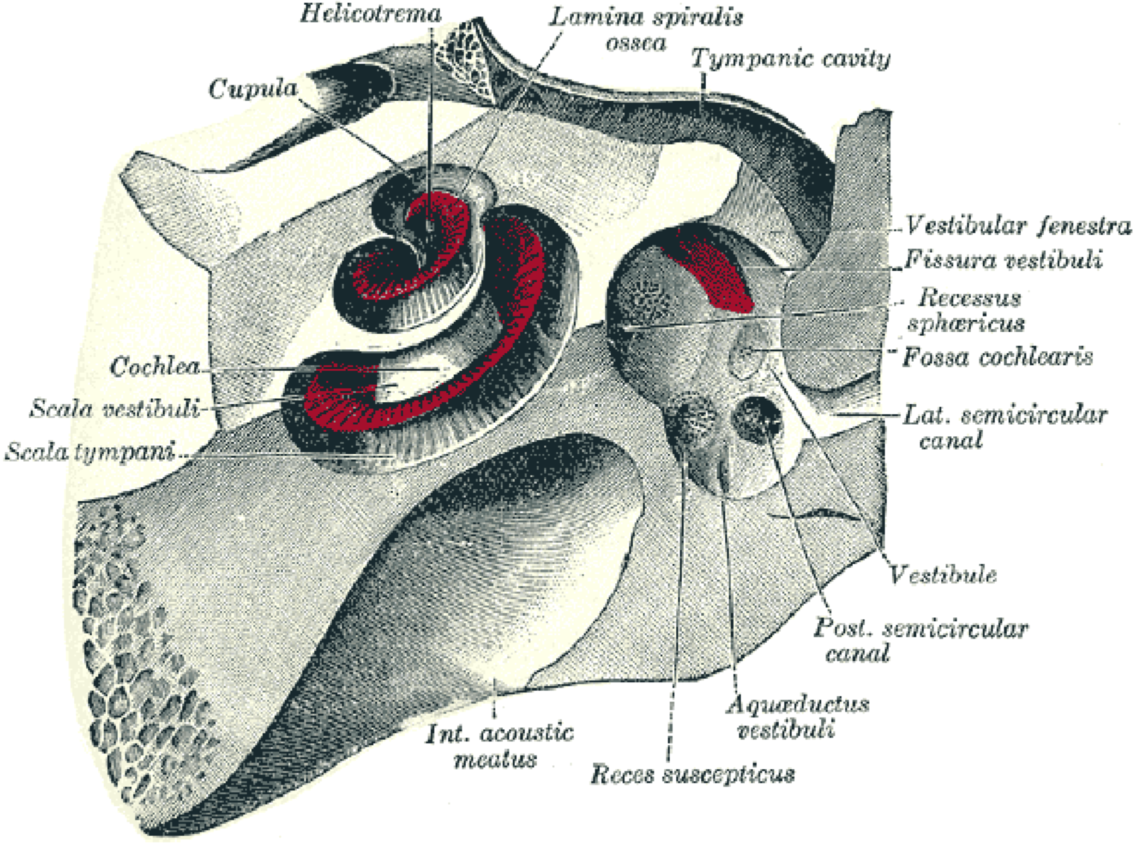




Time domain	Frequency domain
$h(t) = A\delta(t)$ 	$H(f)$ 
$h(t) = A\sum_{n=-\infty}^{-\infty} \delta(t - nT)$ 	$H(f)$ 

Time domain	Frequency domain
$h(t) = \begin{cases} A & \text{if } -T_0 < t < T_0 \\ 0 & \text{otherwise} \end{cases}$	$H(f) = 2AT_0 \frac{\sin(2\pi T_0 f)}{2\pi T_0 f}$
	

Transform in the human cochlea



Video: https://youtu.be/dyenMIuFaUw