Convolution using overlap-and-add

Fast linear filtering using the FFT

FIR

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Linear filtering using the FFT

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] \qquad y[n] = x[n] * h[n]$$

Calculate the discrete convolution using the FFT:

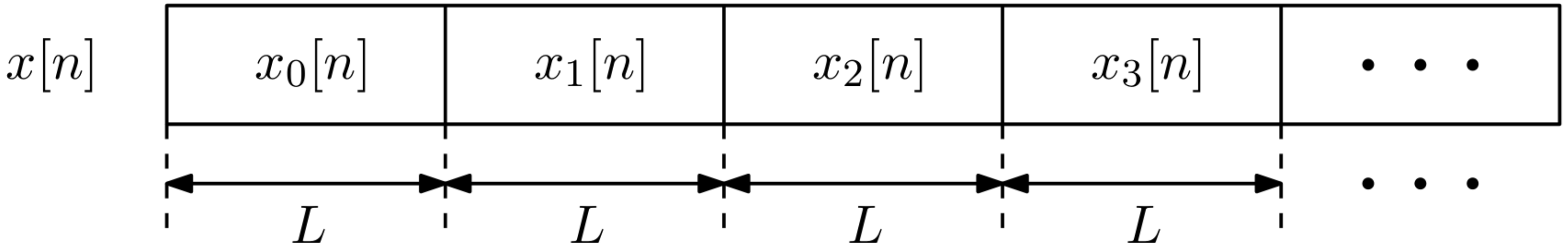
- Length of $x[n]$ is M
- Length of $h[n]$ is P (FIR)

Zero padding

- Zero pad $x[n]$ and $h[n]$ to length $N \geq M + P - 1$
- $y[n] = \mathrm{IFFT}\left\{X_{\mathrm{zp}}[k] \cdot H_{\mathrm{zp}}[k]\right\}$

But $x[n]$ is often streamed in, i.e. not bounded in length

Overlap-and-add



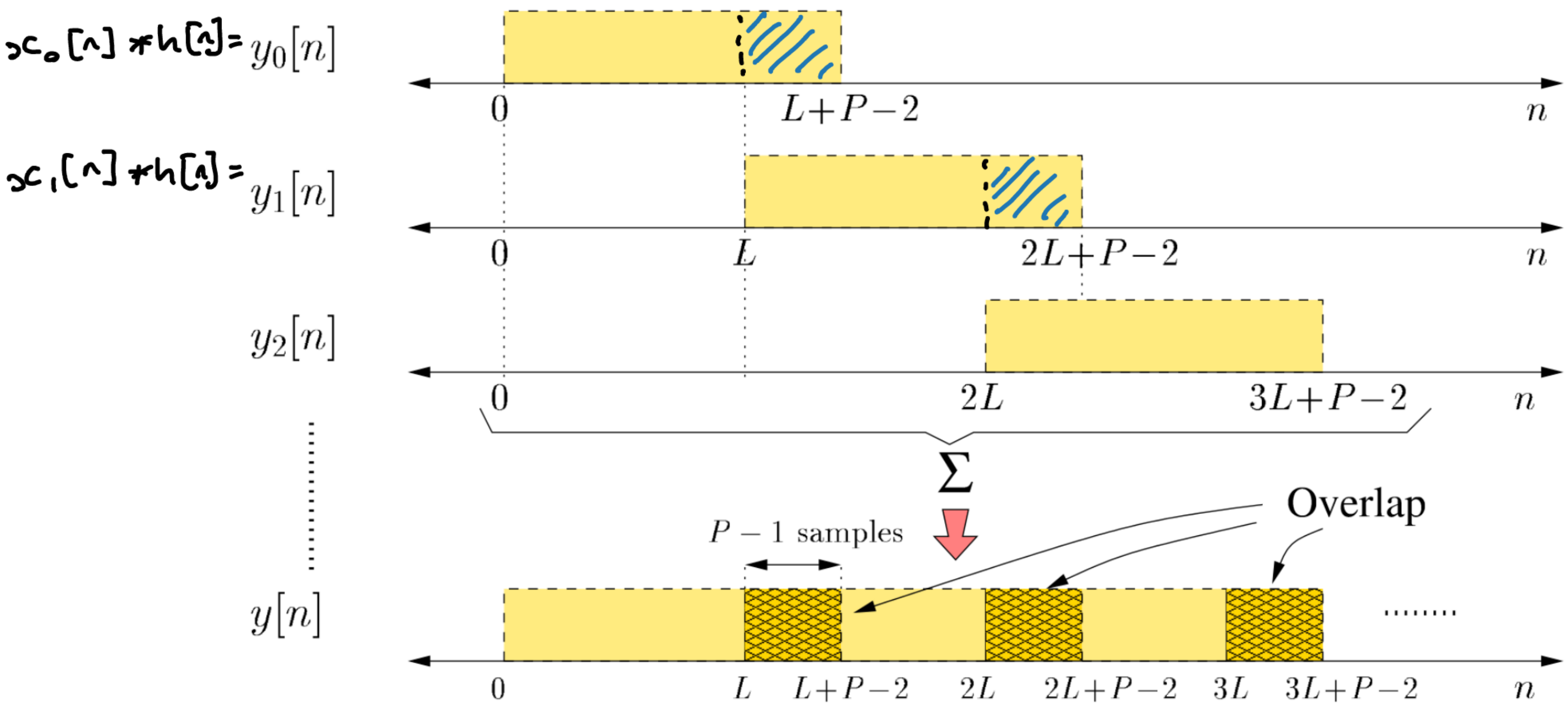
$$x[n] = \sum_{i=0}^{\infty} x_i[n] \qquad \text{where} \qquad x_i[n] = \begin{cases} x[n] & \text{if } n \text{ is in window } i \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} y[n] = h[n] * x[n] &= h[n] * \left(\sum_{i=0}^{\infty} x_i[n] \right) \\ &= \sum_{i=0}^{\infty} h[n] * x_i[n] \\ &= \sum_{i=0}^{\infty} y_i[n] \qquad \text{where} \qquad y_i[n] = h[n] * x_i[n] \end{aligned}$$

$$y_0[n] = \overset{P}{h[n]} * \overset{L}{x_0[n]}$$

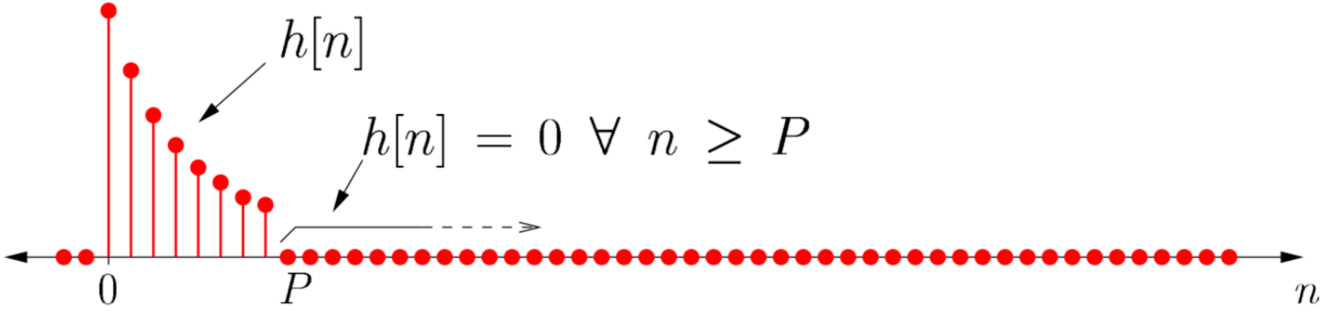
$n = 0$:

$n = ?$:

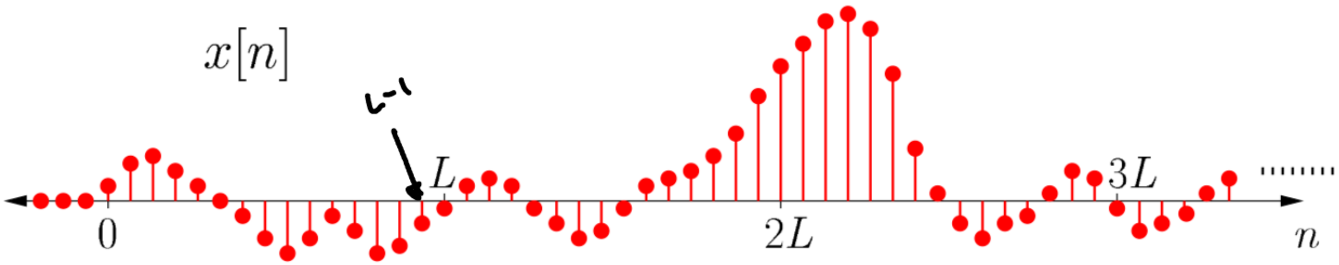


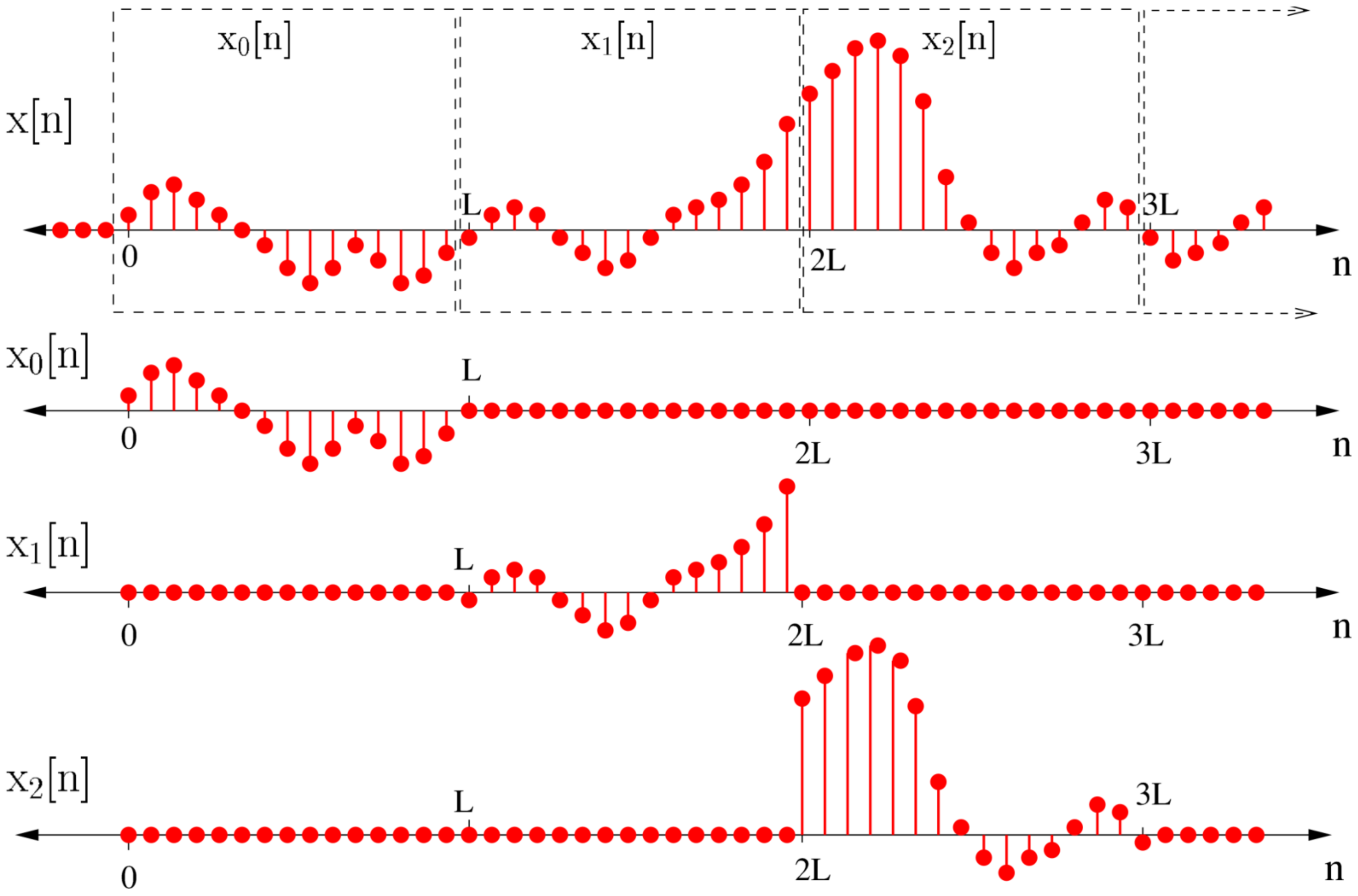
$x_0[n] * h[n] = y_0[n]$
$x_1[n] * h[n] = y_1[n]$
$y_2[n]$
$y[n]$
0
$L+P-2$
n
L
$2L+P-2$
$2L$
$3L+P-2$
Σ
$P-1$ samples
Overlap
$3L$

Overlap-and-add example

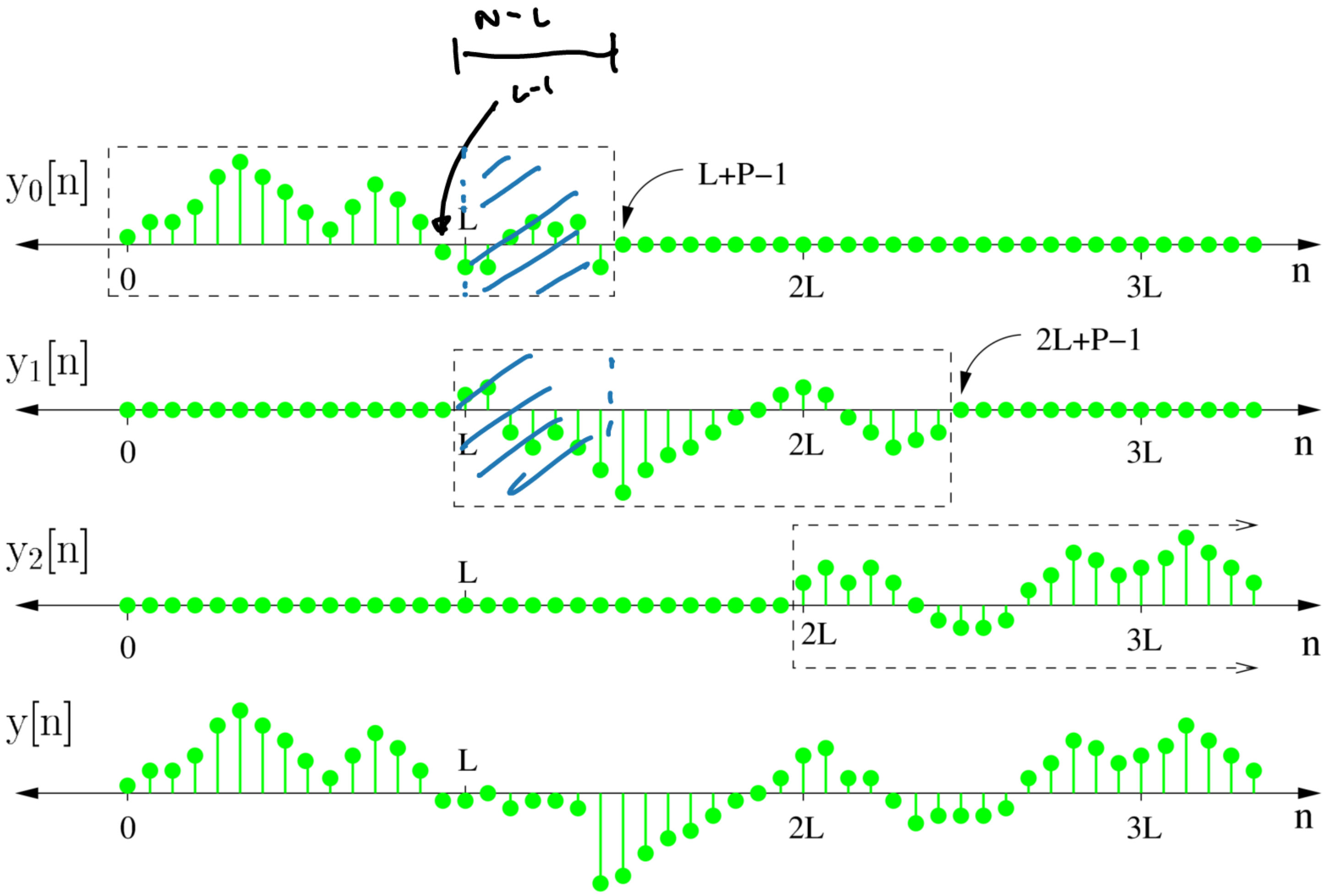


FIR





x[n]
x0[n]
x1[n]
x2[n]
0
L
2L
3L
n



N−L
L−1
y0[n]
L+P−1
L
0
2L
3L
n
y1[n]
2L+P−1
L
0
2L
3L
n
y2[n]
L
0
2L
3L
n
y[n]
L
0
2L
3L
n

Overlap-and-add procedure

- Choose a suitable block length L
- Zero pad $h[n]$ to length $N \geq L + P - 1$
- Calculate $H[k] = \text{FFT}\{h[n]\}$

  zp zp
- For each L-sample block of the input sequence:
  - Zero pad to length N
  - Calculate the FFT
  - Multiply with $H[k]$
    zp
  - Calculate the IFFT
  - Add to $y[n]$, overlapping the last $N - L$ samples
- Final result: $y[n]$

$y_i[n] = x_i[n] * h[n]$

$P \geqslant 64$