Correlation between LTI inputs and outputs

Applications in system identification

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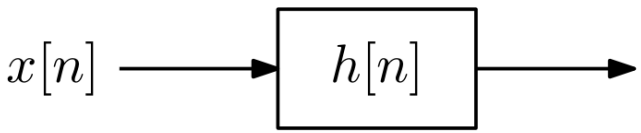
Correlation between inputs and outputs of LTI systems



$$y[n] = x[n] * h[n]$$

Cross-correlation between system output and input:

$$r_{yx}[i] = y[i] * x[-i]$$

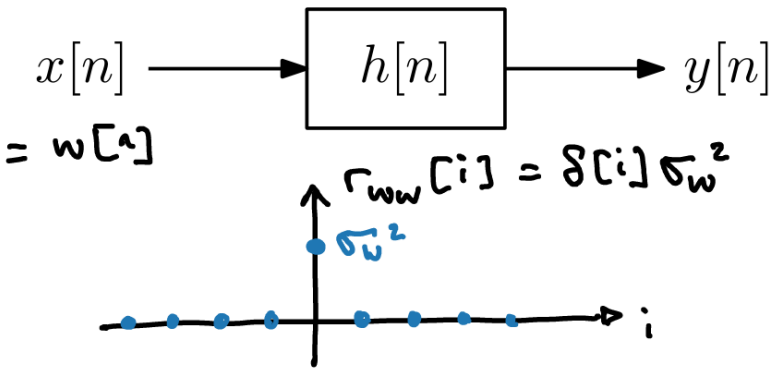
$$= (h[i] * x[i]) * x[-i]$$

$$= h[i] * (x[i] * x[-i])$$

$$= h[i] * r_{xx}[i]$$

$$r_{yy}[i] = r_{hh}[i] * r_{xx}[i]$$

Impulse response estimation

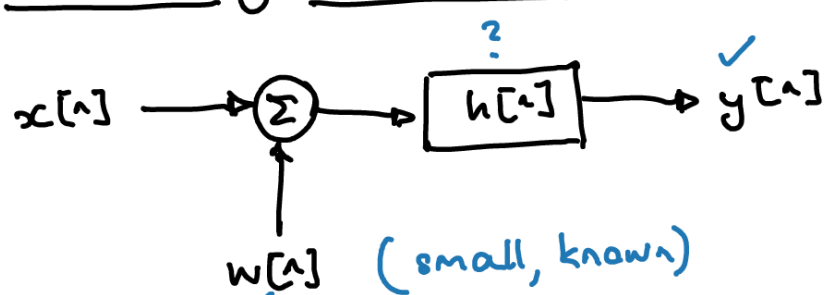


$$y[n] = x[n] * h[n]$$

$$r_{yx}[i] = h[i] * r_{xx}[i]$$

$$\begin{aligned} r_{yw}[i] &= h[i] * r_{ww}[i] \\ &= h[i] * \delta[i]\cdot\sigma_w^2 \\ &= \sigma_w^2 h[i] \end{aligned}$$

Online system ID:



$$y[n] = (w[n] + x[n]) * h[n]$$

$$= x[n] * h[n] + w[n] * h[n]$$

$$r_{yw}[i] = y[i] * w[-i]$$

$$= (x[i] * h[i] + w[i] * h[i]) * w[-i]$$

$z[i]$

$$= x[i] * h[i] * w[-i] + h[i] * w[i] * w[-i]$$

$$= r_{zw}[i] \overset{\approx 0}{+} h[i] * r_{ww}[i]$$

$$= \sigma_w^2 h[i]$$