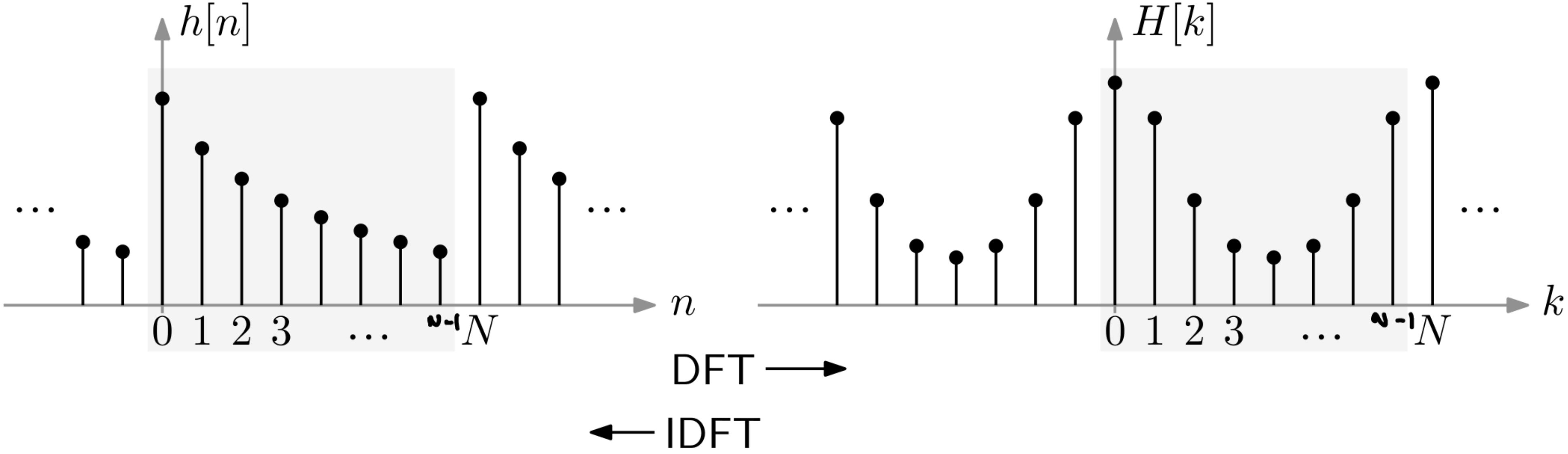
What does the DFT tell us?

Properties and examples

Herman Kamper



np.fft.fft

Properties of the DFT

- Linearity:

$$\mathrm{DFT}\left\{\alpha x[n] + \beta y[n]\right\} = \alpha \mathrm{DFT}\{x[n]\} + \beta \mathrm{DFT}\{y[n]\}$$

- Symmetry:

$$\text{if } \mathrm{DFT}\{h[n]\} = H[k] \text{ then } \mathrm{DFT}\{H[n]\} = N \cdot h[-k] = N \cdot h[N-k]$$

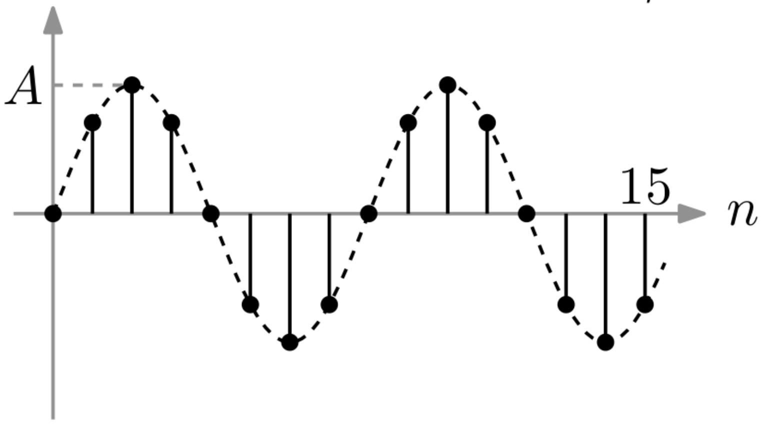
- Even and odd time sequences:
  - If $h[n]$ is even, then $h[n] = h[-n] = h[N-n]$
  - If $h[n]$ is odd, then $h[n] = -h[-n] = -h[N-n]$
  - If $h[n]$ is real, $H[k]$ has an even real and an odd imaginary part

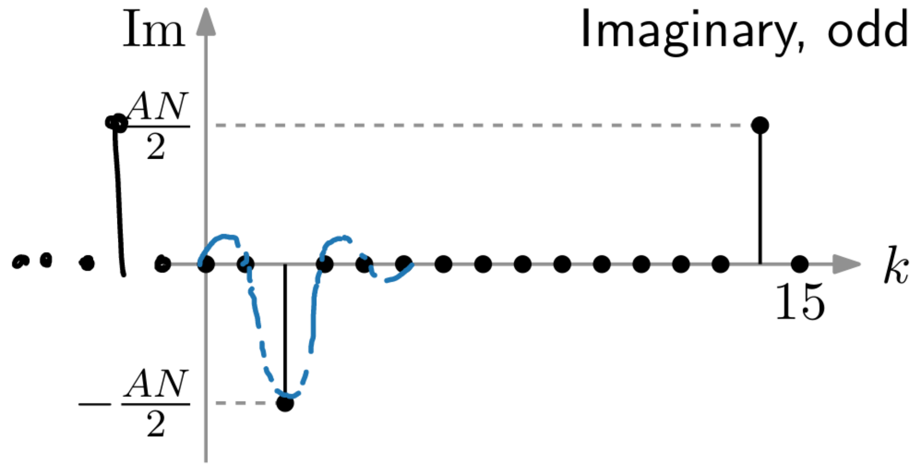
- Time reversal:

$$\mathrm{DFT}\{x[-n]\} = \mathrm{DFT}\{x[N-n]\} = X[N-k] = X[-k]$$

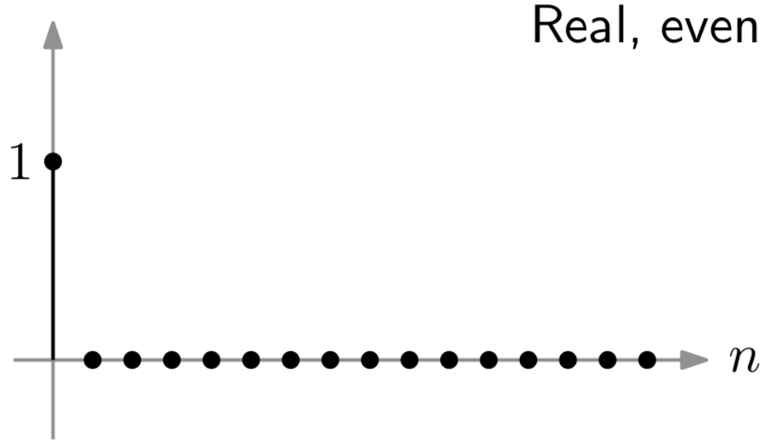
16 - point DFT

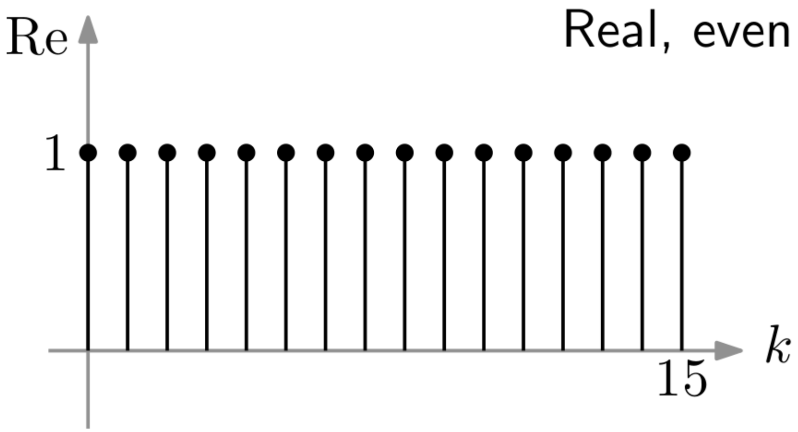
Discrete-time domain	Frequency domain
Real, odd	Imaginary, odd

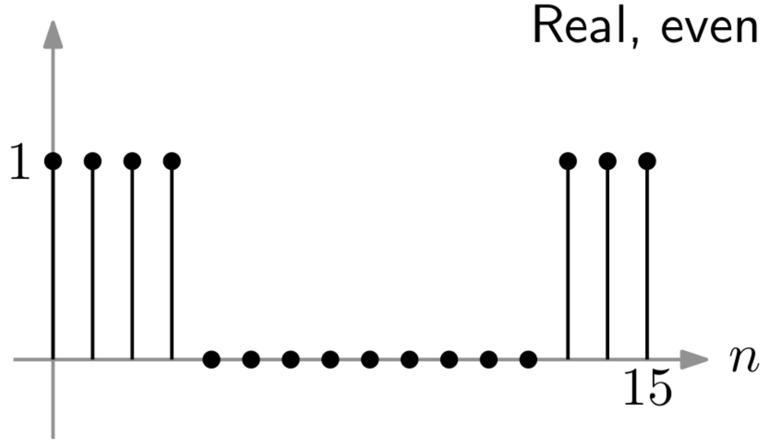


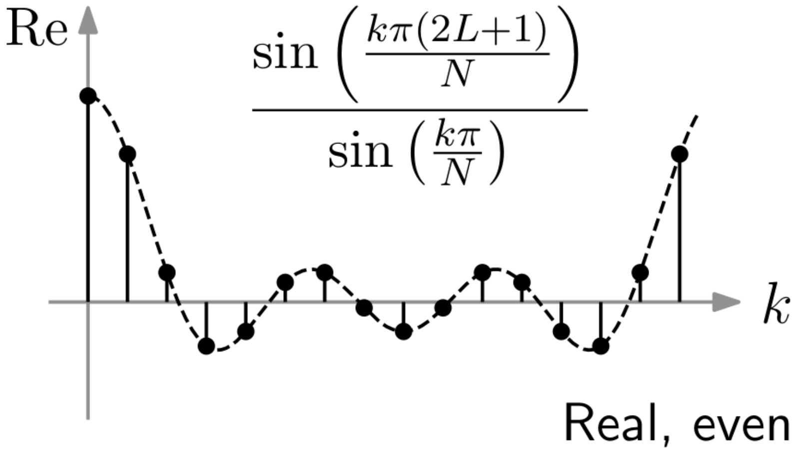


Discrete-time domain	Frequency domain



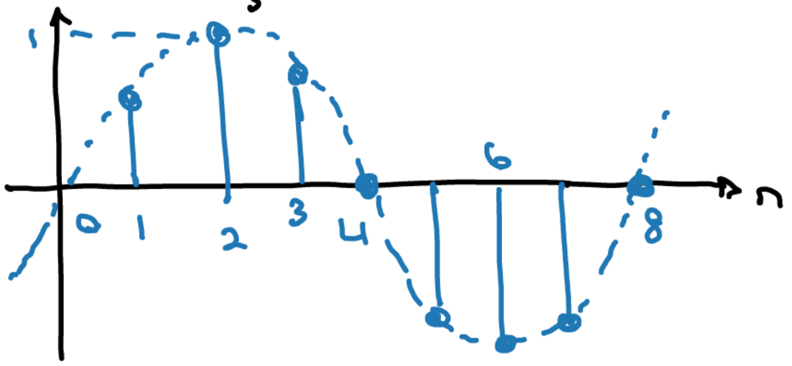






DFT examples

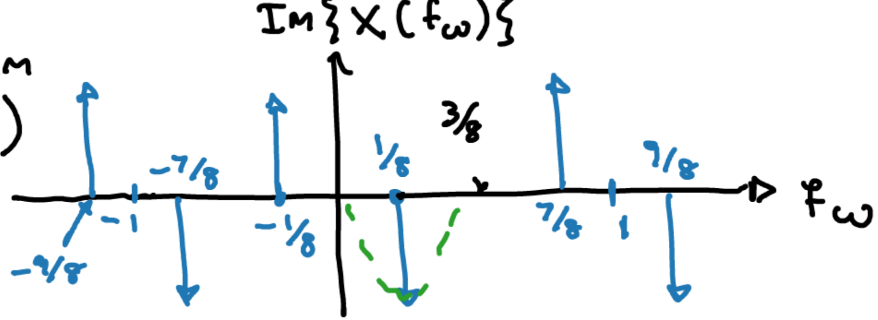
$$\frac{f_0}{f_s} = f_{\omega_0} = \frac{1}{8} \text{ cycles/sample}$$

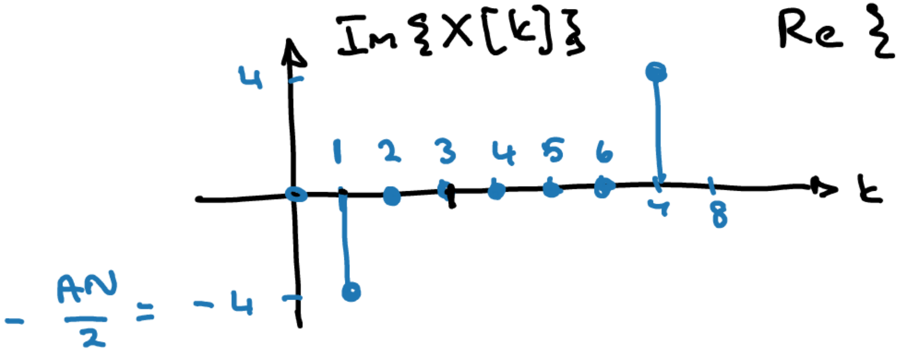


$$x[n] = \sin(2\pi f_{\omega_0} n) = \sin(2\pi \frac{1}{8} n)$$

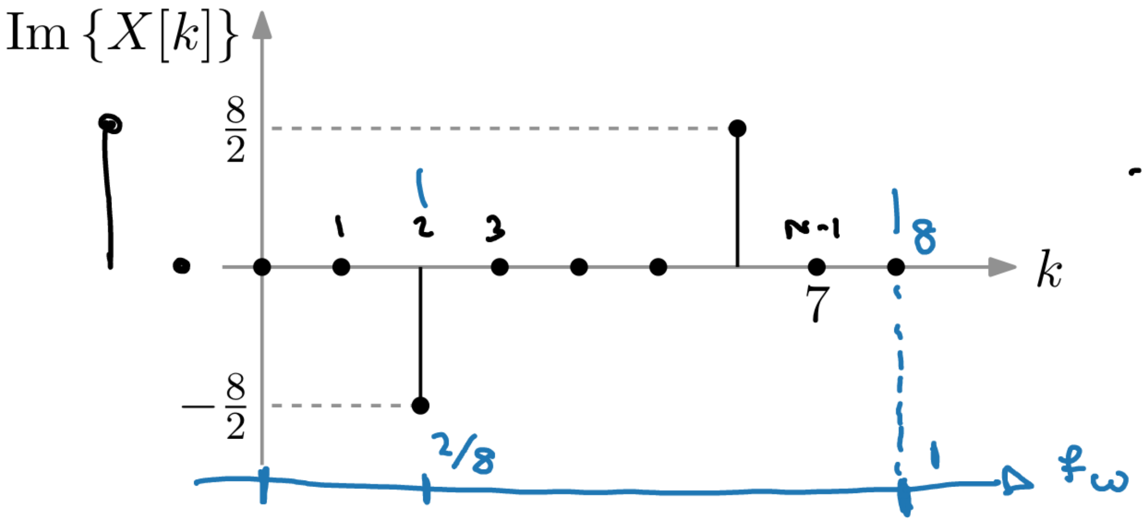
Draw $x[n]$ and its eight-point DFT $X[k]$:

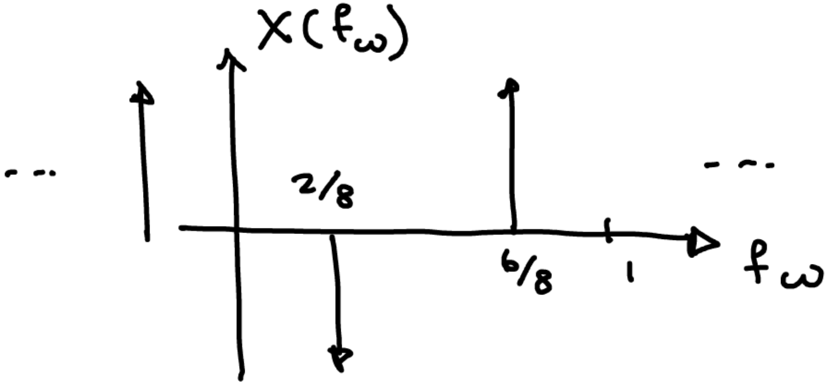
Spectrum (DTFT)





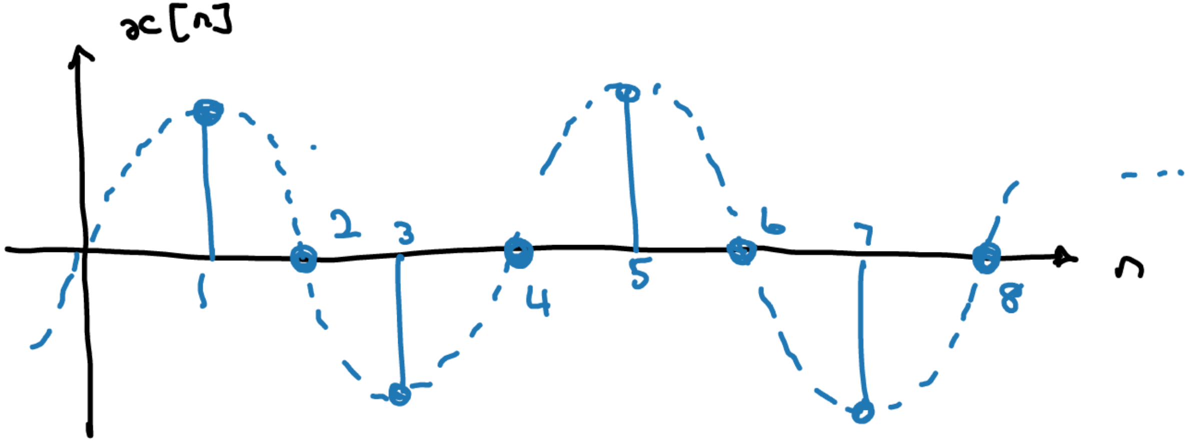
$\mathrm{Re}\{X[k]\} = 0$



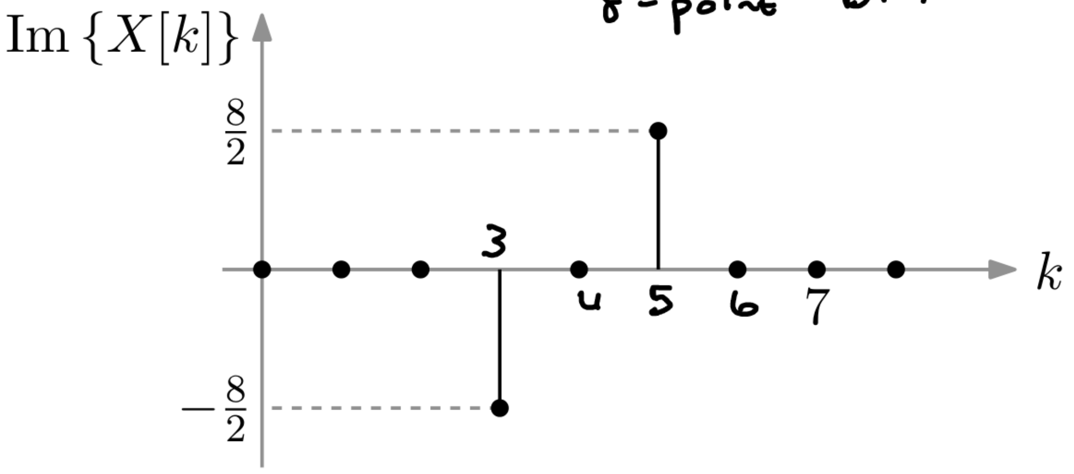


$$f_{\omega 0} = 2/8 = 1/4 \left[\text{cycles}/\text{sample}\right]$$

Draw eight samples of $x[n]$:

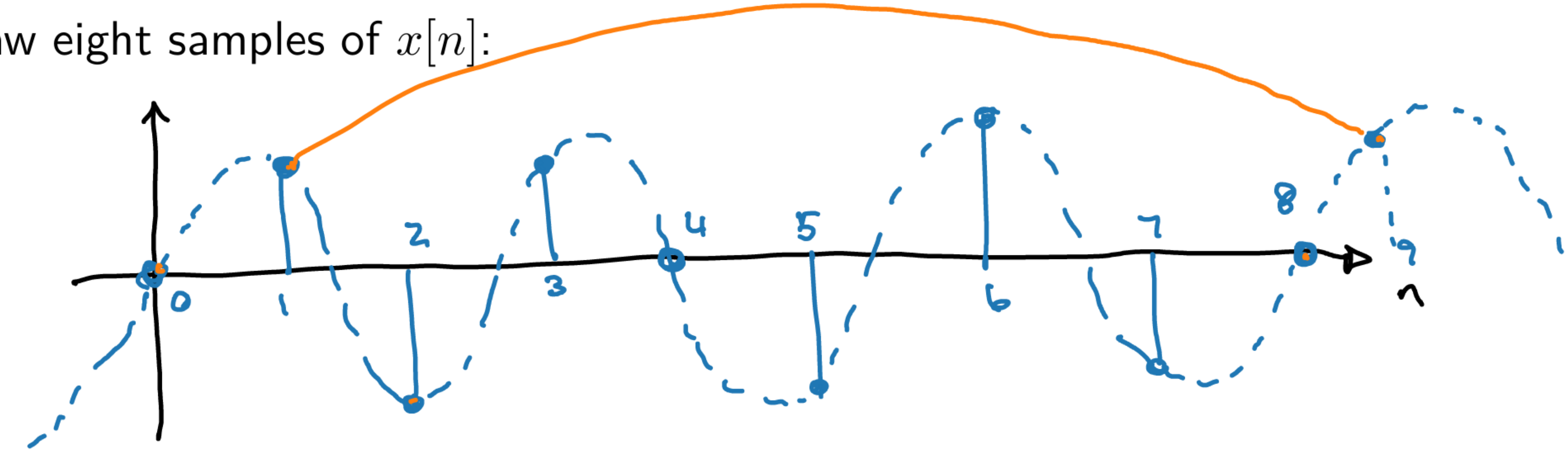


8-point DFT

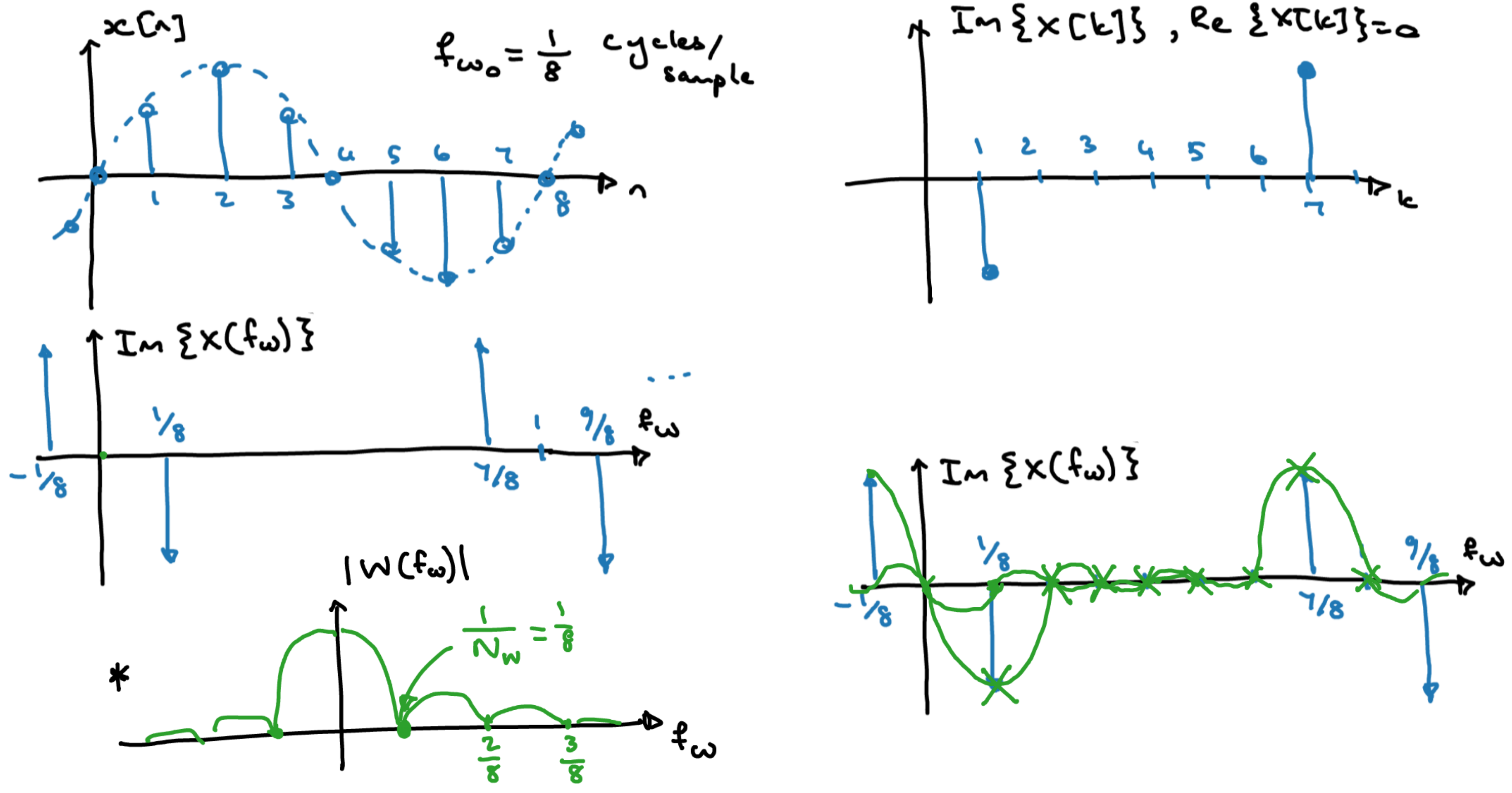


$$f_{\omega_0} = \frac{3}{8} \text{ cycles/sample}$$
$$= \frac{k}{N}$$

Draw eight samples of $x[n]$:



Where did the side lobes go?



Lobe separation

Slides 1.54 – 1.57